Glossary of Terms

a	Chapter 2, impact parameter, $a = r\sin\gamma$; Appendix D, parameter in third-order stationary phase theory.
(a)	Anomalous ray in multipath scenario.
$a_l^{\pm}$	Transmission coefficient of spectral number l; a_l^{+} applies to an outgoing wave; a_l^{-} applies to an incoming wave.
AS	Anti-spoofing.
$\mathrm{Ai}[\hat{y}]$	Airy function of the first kind.
$\mathscr{A}(x, x_o)$	Phase accumulation from turning point at x_o to x in direction perpendicular to Cartesian-stratified plane in thin-film medium.
(b)	Branching ray in multipath scenario.
$b_l^{(j)}$	Reflection coefficient of spectral number l and reflection degree j. For $j \geq 1$, $j-1$ denotes the number of internal reflections of wave before exiting sphere; $j = 0$ denotes external reflection.
B	Chapter 2, half-width of shadow zone; Fresnel aliasing parameter.
$\mathrm{Bi}[\hat{y}]$	Airy function of the second kind.
$\mathscr{B}$	$\mathscr{B} = \mathscr{A}(x_2, x) - \mathscr{A}(x, x_o)$.
c	Speed of light.
$\mathscr{C}(z)$	In-plane phase accumulation along z-direction parallel to the plane of stratification in a Cartesian-stratified thin-film medium.

D	$D = r_{\mathrm{L}} \cos \gamma_{\mathrm{L}}$; essentially the distance of LEO from Earth's limb, generally the reduced distance; length or phase units apply, depending on context.
D	Reduced distance from LEO to thin phase screen, $D^{-1} = D_{\mathrm{G}}^{-1} + D_{\mathrm{L}}^{-1}$; $D_{\mathrm{G}}, D_{\mathrm{L}}$ = GPS, LEO actual limb distances.
D_ν	$D_\nu = \left(\rho^2 - \nu^2\right)^{1/2}$, the tangential distance in spectral number space from sphere of radius ν to point ρ; see Fig. 3-14.
$\boldsymbol{E}$	Electric field vector of electromagnetic wave; $E_{\parallel}$ is the in-plane component; $E_{\perp}$ is cross-plane.
$\hat{E}$	Stopped electric field with nominal RF removed.
$\hat{E}[\omega]$	Fourier transform of $\hat{E}$.
f	Frequency of harmonic wave.
f_d	Excess Doppler due to refraction.
$F[x]$	Chapter 2 and Appendix D, $F[x] = \left(1 - \mathrm{erf}\left[x^2\right]\right)\exp\left(x^2\right)$; Chapter 3, $F[x] = \left(1 - x^2\right)^{-1/2} \tan^{-1}\left(x^{-2} - 1\right)^{1/2}$; Chapter 6, spectral function, proportional to the bending angle.
$f(x_1, x_2)$	Off-diagonal element in the 2×2 characteristic matrix $\boldsymbol{M}[x_1, x_2]$ for the parallel component of a wave in a stratified thin-film medium.
$F(x_1, x_2)$	On-diagonal element in the 2×2 characteristic matrix $\boldsymbol{M}[x_1, x_2]$ for the parallel component of a wave in a stratified thin-film medium.
$\mathscr{F}$	Vertical radius of the first Fresnel zone.
$g_l(\rho)$	Phase-rate function for the lth spectral coefficient at position ρ.
$g(\hat{y})$	Asymptotic version of $g_l(\rho)$.
$g(x_1, x_2)$	On-diagonal element in the 2×2 characteristic matrix $\boldsymbol{M}[x_1, x_2]$ for the $\perp$ component of a wave in a stratified thin-film medium.
$G(x_1, x_2)$	Off-diagonal element in the 2×2 characteristic matrix $\boldsymbol{M}[x_1, x_2]$ for the $\perp$ component of a wave in a stratified thin-film medium.
$G[\rho, \nu]$	Refractive gradient-induced phase accumulation in the lth spectral coefficient, $a_l^{\pm}$, at the position $\rho = nkr$, $\nu = l + 1/2$.
$G[\mu]$	$G[\mu] = \left(1 - \mu^2\right)^{1/2} \tan^{-1}\left(\mu^{-2} - 1\right)^{1/2}$.

h	Chapter 2, thin-screen altitude in length units; Chapter 3, penetration depth in Fresnel formulas.
h_S	Thin-screen altitude of the ray impact parameter, in phase units.
h_{LG}	Thin-screen altitude of straight line between an occulted GPS satellite and a LEO.
$h(1)$	Lower boundary in thin phase screen for multipath.
$h(2)$	Upper boundary in thin phase screen for multipath.
$\boldsymbol{H}$	Magnetic field vector of electromagnetic wave.
H	Scale height in exponential refractivity distribution; H_p is pressure scale height; H_p is density scale height.
H_S	Refractivity scale height in the transition zone.
H_w	$1-\sigma$ half-width of a Gaussian refractivity distribution.
$H_l^{\pm}(x)$	Hankel function of first (+) and second (–) kinds.
$\mathscr{H}[\mathrm{x}]$	Heaviside function.
$\imath$	Angle of incidence at a reflecting boundary.
I	Chapter 3, amplitude of incident ray in Fresnel formulas; Chapter 3, scattering from a sphere at LEO position minus diffraction from a knife-edge; Chapter 6, angle between the LEO orbit plane and the plane of incidence.
$I(\rho_{\mathrm{L}},\theta_{\mathrm{L}})$	Electric field at the LEO from a reflecting sphere.
$J(\rho_{\mathrm{L}},\theta_{\mathrm{L}})$	Electric field at the LEO from the direct wave with a reflecting sphere; total field $E = I + J$.
J	Knife-edge diffraction integral.
$J_l(x)$	Bessel function of the first kind of order l.
k	Wave number of harmonic wave $k = 2\pi/\lambda$.
K_H	Sensitivity of refractive bending angle to change in scale height.
K_γ	Sensitivity of refractive bending angle to change in lapse rate.
K_ρ	$K_\rho = (\rho/2)^{1/3}$; $K_\nu = (\nu/2)^{1/3}$; $\rho = nkr$, $\nu = l + 1/2$.
l	Spectral number, an integer.
L1	Chapter 1, GPS carrier at 1.575 GHz; Chapter 2, phase in range units.
L2	Chapter 1, GPS carrier at 1.228 GHz; Chapter 2, phase in range units.

(m)	Main ray in multipath scenario.
M	Sample size number.
$M^{@}$	Fresnel aliasing threshhold for discrete sampling.
$M[y]$	Modulus of the derivatives of the Airy functions.
$\boldsymbol{M}[x_1, x_2]$	Unitary state transition matrix for the electromagnetic field in a stratified refracting medium.
n	Index of refraction function in a refracting medium.
n_e	Electron number density.
N	Refractivity $\times 10^{-6}$; $N = n - 1$.
p	Pressure.
p_k	Generalized parameter in bending-angle function.
$P_l^m(x)$	Legendre polynomial of order l and degree m.
r	Radial coordinate; r_{L} applies to the LEO; r_{G} to the GPS satellite.
r_o	Geocentric distance of spherical boundary surface.
$\mathscr{r}$	Angle of reflection at a reflecting boundary.
R	Amplitude of the reflected wave from Fresnel formulas.
R_E	Radius of the Earth.
$\mathscr{R}$	Ratio of amplitudes at the LEO of the main ray and caustic ray.
s	Arc length along ray.
S_l	Aggregate scattering coefficient for spectral number l.
$S_l^{(j)}$	Scattering coefficient for a wave with $j-1$ internal reflections for $j \geq 1$; $S_l^{(0)}$ is the coefficient for an external reflected wave.
$\boldsymbol{S}$	Poynting vector; $\boldsymbol{S}^{(i)}$ applies to the incident wave; $\boldsymbol{S}^{(s)}$ applies to the scattered wave.
$\mathscr{S}$	Eikonal function; eikonal equation is $\lvert \nabla \mathscr{S} \rvert = n(x, y, z)$.
t	Time.
T	Time interval spanned by data set; Chapter 3, amplitude of the transmitted wave in Fresnel formulas; Chapter 2 and Appendix A, temperature.
u	Chapter 2, dimensionless thin screen altitude, $u = h(2/\lambda D)^{1/2}$; Chapter 5, dimensionless radial coordinate, $u = kr$.
$U(x)$	Solution to the electromagnetic wave equations for the transverse component of the field in a stratified medium.

$V(x)$	Solution to wave equations for the in-plane tangential component of the field in a stratified medium.
$W(x)$	Solution to wave equations for the in-plane $\perp$ component of the field in a stratified medium.
$W(\rho, \rho_*)$	Weighting function in the wave theory analog of the Abel transform.
$V_\perp$	Component of the geocentric radial velocity of the tangency point of the GPS–LEO straight line in the propagation plane.
$\mathscr{W}_l, \mathscr{W}_l^{\pm}$	Mie scattering functions in transfer equations across a boundary.
x	Chapter 3, in-plane Cartesian coordinate $\perp$ to $\boldsymbol{S}^{(i)}$; also $x = kr$; Chapter 4, in-plane Cartesian coordinate $\perp$ to the plane of stratification.
y	Argument of the Airy functions in a homogeneous medium; Chapter 3 and Chapter 5, cross-plane Cartesian coordinate $\perp$ to $\boldsymbol{S}^{(i)}$; Chapter 4, cross-plane coordinate parallel to the plane of stratification.
$\hat{y}$	Argument of the Airy functions in a refracting medium; $\hat{y} = \nu^{2/3}\zeta[\nu / \rho]$, $\rho = nkr$, $\nu = l + 1/2$.
$\hat{y}^\dagger$	$\hat{y}^\dagger = 0.441\cdots$, the zero point of $g_l(\rho)$.
$Y_l(x)$	Bessel function of the second kind of order l.
z	Chapter 2, deviation of the ray path from a straight line; Chapter 3 and Chapter 5, in-plane Cartesian coordinate parallel to $\boldsymbol{S}^{(i)}$; Chapter 4, in-plane coordinate parallel to the plane of stratification; Chapter 5, argument of the Airy function in third-order theory.
α	Chapter 2, refractive bending angle at the LEO.
$\tilde{\alpha}$	$\tilde{\alpha}(\rho, \rho_*)$ is the cumulative bending angle on an incoming ray at position (r, θ) with $\rho = nkr$ and with an impact parameter value of ρ_*. For a spherical symmetric medium, the cumulative bending angle on an outgoing ray is $2\tilde{\alpha}(\rho_*, \rho_*) - \tilde{\alpha}(\rho, \rho_*)$.
α_{L}	Refractive bending angle at the LEO; $\alpha_{\mathrm{L}}(\rho_*) = 2\tilde{\alpha}(\rho_*, \rho_*)$; ρ_* obtained from Bouguer's law.
β	$\beta = -r(dn / dr) / n$; β is the ratio of the radius of curvature of the iso-refractivity surface to the radius of curvature of the ray.

γ	Chapter 2 and Appendix A, normalized lapse rate, $\gamma = (dT/dr)/T$; Chapter 4, $\gamma = \left(2n_0 n'/k\right)^{1/3}$ in a Cartesian stratified Airy layer; Appendix A and Chapter 5, the angle between the radius vector and the tangent vector of the ray; $\gamma = \theta + \alpha$ when r_{G} is infinite.
$\Gamma[\mathrm{x}]$	Gamma function.
$\Gamma(\hat{y})$	$d\Gamma / d\hat{y} = 3g(\hat{y})$.
δ	$\delta = \theta_{\nu *} - \theta$ equals the refractive bending angle to first order in *N*.
$\delta(x)$	Dirac delta function.
ε	Chapter 2, in the thin-screen model, $\varepsilon(h)$ is the phase perturbation due to discontinuity in a refraction parameter; Chapters 3, 4, and 5, dielectric coefficient in a refracting medium, assumed linear; Chapter 6, azimuthal angle of the apparent direction to the occulted GPS satellite as seen from the LEO.
ζ	Defocusing factor $\zeta^{-1} = 1 - Dd\alpha / d\rho_*$.
$\zeta[\nu/\rho]$	Relates the argument $\hat{y}$ of the Airy functions to ν and ρ in the Bessel functions, $\hat{y} = \nu^{2/3}\zeta[\nu/\rho]$.
η	Auxiliary parameter in multipath altitude separation.
θ	Geocentric central angle in the orbit plane relative to the z-axis; θ_{L} applies to the LEO; θ_{G} applies to the GPS satellite.
$\tilde{\theta}_{\mathrm{L}}$	Geocentric central angle of the LEO relative to the z-axis in the plane of propagation; see Fig. 6-8(a).
θ_ν	$\theta_\nu = \sin^{-1}(\nu/\rho)$; see Fig. 3-14; θ_ν is the “angle of incidence” of the νth spectral component in a spherical framework.
κ	Chapter 2, the coefficient relating refractivity to n_e, $\kappa = 40.3\,\mathrm{m}^3/\mathrm{s}^2/e$; Chapter 3, the extinction coefficient in an absorbing medium.
λ	Wavelength of the harmonic wave, $\lambda = 2\pi/k$.
μ	Chapters 3, 4, and 5, magnetic permeability of the refracting medium; Chapter 3, $\mu = \nu/\rho$.
ν	Fractional spectral number, $\nu = l + 1/2$; Chapter 2, $\nu = (r_o - r)/r_o$.
ν^*	Stationary phase value of fractional spectral number ν.
ξ	Parameter in multipath altitude separation, Appendix C.

$\xi_l^{\pm}(x)$	Spherical Hankel function of the first (+) and second (–) kinds, $\xi_l^{\pm} = \sqrt{\pi x / 2} H_\nu^{\pm}(x) = \sqrt{\pi x / 2}(J_\nu(x) \pm iY_\nu(x))$.
Π	Scalar potential for electromagnetic wave; ${}^{\mathrm{e}}\Pi$ is electric potential; ${}^{\mathrm{m}}\Pi$ is magnetic potential.
ρ	Radial coordinate $\rho = krn(r)$; Appendix A, mass density.
$\tilde{\rho}$	$\tilde{\rho} = n_{\mathrm{A}} kr$; n_{A} is a constant.
ρ_{L}	Radial position of the LEO in spectral number space.
ρ_*	Ray path impact parameter.
$\rho_{\mathscr{C}}$	Impact parameter at a caustic contact point.
$\rho^\dagger$	$\rho^\dagger = \nu - \hat{y}^\dagger K_{\rho^\dagger}$, a stationary point for $G[\rho, \nu]$ with respect to ρ.
$\rho^{(0)}$	Wave theory analog of Fresnel reflection coefficient.
σ	Chapter 2, $\sigma = \sqrt{(r_o - r) / H_{p_o}}$; Appendix E, parameter in the electron density model; Chapter 5 and Appendix A, $\Delta\sigma$ is the $\perp$ displacement between two rays with an impact parameter difference $\Delta\rho_*$; Chapter 5, parameter in the generalized Fresnel transform of $\mathrm{Ai}[y]$.
σ_+	Chapter 2, accounts for ray path bending in the case of a discontinuous refractivity, $\sigma_+^2 = \sigma^2 + \beta\Delta N / N$.
τ	Time interval; Chapter 5, parameter in the generalized Fresnel transform of $\mathrm{Ai}[y]$.
ϕ	Azimuthal angular coordinate around the z-axis.
$\varphi(h_S)$	Stationary phase profile embedded in the thin phase screen.
φ	Angle of incidence in Fresnel formulas.
$\Phi(h_S, h_{\mathrm{LG}})$	Thin-screen Fresnel phase $= \varphi(h_S) + (h_S - h_{\mathrm{LG}})^2 / (2D)$.
$\Phi^{\pm}$	Chapters 3 and 5, geometric phase delay to the LEO from the tangency point on the near side (–) or far side (+) of the sphere of radius ν.
Φ^*	Chapter 5, $\Phi^* = \Psi(\pm,-)\big\rvert_{\nu=\nu^*} \pm \pi/4$, the stationary phase at point (r,θ) from wave theory.
$\chi_l(x)$	Spherical Bessel function of the second kind of order l, $\chi_l(x) = \sqrt{\pi x / 2} Y_{l+1/2}(x)$.

$\hat{\mathrm{X}}$	$\hat{\mathrm{X}} = 2(-\hat{y})^{3/2}/3 + \pi/4$, variable in the negative argument asymptotic forms for the Airy functions.
$\psi(h_{\mathrm{LG}})$	Phase of wave at the LEO in the thin screen model.
$\psi_l(x)$	Spherical Bessel function of the first kind of order l, $\psi_l(x) = \sqrt{\pi x/2}J_{l+1/2}(x)$.
$\Psi(\pm,\pm)$	Chapter 5, spectral density function for the phase of the νth spectral component of the wave at the point (r,θ).
$(\pm,\pm)$	The four combinations $(\pm,\pm)$ yield the phase spectral densities $\Psi(\pm,\pm)$ for waves passing through either the near or the far side of a sphere, and for either incoming or outgoing waves. When $r_o/\lambda >> 1$, only the near-side combinations $(\pm,-)$ contribute to the field at (r,θ) when $0 << \theta << \pi$, and in this case only the outgoing combination $(+,-)$ applies at the LEO.
$\Psi^{(j)}$	Chapter 3, reflection angle from geometric optics of an exit ray after j – 1 internal reflections; Chapter 3, phase of degree j scattering phasor plus near-side geometric delay $\Psi^{(j)} = 2\Omega^{(j)} + \Phi^{-}$.
$\Psi^{(S)}$	Chapter 3, phase of aggregate scattering plus near-side geometric phase delay at (r,θ), $\Psi^{(S)} = \Psi^{(S)}(x,\theta,N,x_o,\nu) = 2\Omega^{(S)} + \Phi^{-}$.
ω	Angular frequency of the harmonic wave, $\omega = ck$.
ϖ	Chapter 4, $\varpi(x) = \left(n(x)^2 - n_o^2\right)^{1/2}$ in a Cartesian stratified Airy layer; $n_o = n(x_o) = \text{constant}$; x_o is a turning point.
$\Omega^{(j)}$	Chapter 3, phase of *j*th degree scattering coefficient $S_l^{(j)}$; $2\,\Omega^{(S)}$ is the phase of S_l; Ω_A is the asymptotic form of $\Omega^{(S)}$ for $\nu << \rho$.